A data-driven method for understanding and increasing 3-point shooting percentage

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Abstract

Although 3-point shooting is an essential aspect of winning games, shooting percentages have remained stagnant for decades. Here, we analyze 6 shooter factors from over 1.1 million 3-point shots captured by the Noah shooting system to quantitatively define high percentage shooting and shooter improvement. We find significant associations between all of these 6 shooter factors and shooting percentage. Furthermore, we use the interaction of these factors to define the region in the hoop where shots are guaranteed to score. Of the 6 factors, 4 are directly actionable using new technologies for instant feedback. We use machine learning to predict shooting percentage within 1.5% using only these 4 factors as input. Finally, we grouped players by their proficiency at these 4 factors and show case studies about the dissimilar training approaches that will lead to optimal improvement for two of these groups.

1. Introduction

In the 2015-2016 season, NBA teams averaged 24.1 3-point attempts per game, with the Golden State Warriors topping the league with an unprecedented 31 attempts per game [1,2]. In the last 15 years, the number of 3-point attempts across the league has rapidly increased; however, the average 3-point shooting percentage has remained constant over the past 20 years [1]. With the increased importance of the 3-point shot to winning games, why have players been unable to increase their yield?

In the past, a number of theoretical papers have been written which propose ways of increasing yield from basketball shots [3,4,5], and some methods have been proposed for collecting and analyzing multi-factor 3-point shot data [6,7].

In this paper we move from theoretical ideas all the way to actionable training methods based upon proven techniques using instant verbal feedback. The foundation for this paper is the measurement of 1.1 million 3-point shots from over 160 players, gathering information on 6 ball dynamics factors in addition to player identification, shot location and make-miss. We use these factors to quantitatively define the zone in the hoop plane where shots score, and then select the 4 trainable factors which accurately predict shooting percentage. Finally, we cluster players based on the 4 factors allowing each individual player to take the most efficient path to improve their 3-point shooting percentage using instant verbal feedback training techniques.
2. Data collection and attribute descriptions

2.1 Data collection

A Noah shooting system was placed approximately 13 feet above the basketball hoop to gather real time data on each shot (Figure 1). This sensor system collected the shot data of 4 NBA teams, 1 WNBA team, 4 college teams and 4 high school teams allowing us to collect data on about 1.1 million 3-point shots in 2016.

2.2 Shot attribute descriptions

For each shot taken, the sensor system collects data about how and where the basketball shot enters the plane of the hoop. We analyze three attributes of shot entry: Left-Right, Depth and Angle. Since 3-point shots are taken from all positions around the 3-point curve, these shot entry attributes are measured from the perspective of the shooter; the point on the hoop closest to the shooter is always defined as the front of the hoop. In the figures, the front of the hoop is shown with a green dot.

Left-Right is the left to right deviation of the shot at the hoop. First an imaginary line is drawn from the shooter through the center of the hoop, defining a perfectly straight shot. Then the flight of the shot is recorded as the parabola drawn through the center of the ball during the flight of the shot. Next the sensor measures the point where the ball flight parabola crosses the plane of the hoop. The Left-Right attribute is the distance between the straight shot line and the point where the ball actually crosses the hoop plane (Figure 2A). Since the hoop has a 9” radius, a shot which lands exactly on the leftmost part of the hoop from the perspective of the shooter has a Left-Right value of -9”, a straight shot has a Left-Right value of 0” and a shot which lands on the rightmost part of the hoop has a Left-Right value of +9”.

Figure 2: Visualizations of shot attributes at the plane of the hoop - A) Left-Right attribute, B) Depth attribute and C) Angle attribute.
Depth is the entry depth of the shot into the hoop. First the front of the hoop is defined as the point on the hoop closest to the shooter. Then the tangent to the hoop is drawn through the front of the hoop point. Next the sensor measures the point where the ball flight parabola actually crosses the plane of the hoop. The Depth attribute is the distance between the front of the hoop tangent line and the point where the ball crosses the hoop plane (Figure 2B). Since the hoop has a diameter of 18", a straight shot which lands directly on the front of the hoop from the perspective of the shooter has a Depth value of 0” and a straight shot which lands directly on the back of the hoop has a Depth value of 18”.

Angle is the entry angle of the shot into the hoop. First the flight of the shot is recorded as the parabola drawn through the center of the ball during the flight of the shot. Then a line is drawn as a tangent to the ball flight parabola as it crosses the hoop plane. The Angle attribute is the angle between the hoop plane and the tangent to the ball flight parabola at the hoop plane (Figure 2C). For example, a relatively flat shot can have an Angle value of 36° and a relatively high arcing shot can have an Angle value of 55°.

In this paper, we use these three attributes of shot entry to describe 6 distinct factors of shooters.

3. Six shooter factors of high percentage shooting

3.1 Left-Right value and Left-Right consistency

The Left-Right value of the 1.1 million 3-point shots captured follows a symmetric distribution around the center of the hoop (Figure 3A). Shots that are more precise to the center of the hoop have a higher probability of scoring regardless of any other factor. Shooters who have a systematic Left-Right value error can easily see this problem with their eyes and correct their future shots accordingly, thus it is not surprising that even though individuals may have a left or right tendency, the overall population had a median Left-Right value of 0”.

Next we measured the Left-Right consistency of individual shooters. We defined consistency as the average absolute deviation from the median. We decided to use absolute deviation rather than the traditional squared to minimize the impact of outliers. Shooters with consistent Left-Right aim tend to make a higher percentage of shots than shooters with inconsistent Left-Right aim (Spearman rho = -0.88, Figure 3A).

Figure 3: A) We show the distribution of the Left-Right values for all 3-point shots measured. The color of each bar corresponds to the make percentage at each Left-Right value. B) We show the correlation between Left-Right consistency and 3-point shooting percentage for individual shooters.
3.2 Depth value and Depth consistency

Similar to the Left-Right value, the Depth value of the 1.1 million 3-point shot population is evenly distributed around the center of the basket at 9” (Figure 4A). However, unlike Left-Right, the highest shooting percentage Depth value is not the center of the basket. A higher percentage of shots are made when they enter the hoop at 11” behind the front of the rim, 2” deeper than the center. (See Highest Percentage arrow in Figure 4A.) The whole population across skill levels from high school through the NBA are consistently practicing suboptimal Depth values. This striking information will be discussed more later in the paper.

We then measured Depth consistency for individual shooters. As with Left-Right consistency, we defined consistency as the average absolute deviation from the median. Regardless of Depth median and other factors, we found a very strong correlation between a player's Depth consistency and the player's shooting percentage (Spearman rho = -0.89, Figure 4B).

3.3 Angle value and Angle consistency-

The mode of the Angle values in the 1.1 million 3-point shot population is 45° with a slightly asymmetrical distribution (Figure 5A). While the shooting percentage of shots in the mid-40s is higher than the surrounding Angles, the decrease in shooting percentage is not the precipitous drop we observed in the Left-Right value and the Depth value.

However, when individual players are scored based on the consistency of Angle, a clear correlation emerges between the Angle consistency and shooting percentage (Spearman rho=-0.63, Figure 5B). Angle consistency is an important factor as will be discussed in more detail later in this paper.
4. Relationships among shooter factors

4.1 Angle consistency is correlated with Depth consistency

We observe a high correlation (Spearman rho = -0.63) between Angle consistency and the Depth consistency for individual shooters (Figure 6). Shooters with high shooting percentages tend to have good Angle consistency and good Depth consistency.

4.2 Angle median is a key determinant of Depth median and Depth consistency

The data from the 1.1 million shot population revealed a surprising but explainable influence which Angle median had on both Depth median and Depth consistency. Figures 7A and 7B show the same data but with the y-axis expanded in 7B.

Depth median - For the population, a shot goes deepest in the basket when shot at an Angle value of 42°. The Depth median value at 36° is about 8" and increases smoothly to about 10" at 42° and then decreases smoothly to about 5" at 55°. Further, the incremental change in Depth value for each change in Angle value is very small around 42°, but the incremental change in Depth value increases faster the further you move away from 42°.

Depth consistency - For the population, Depth is most consistent at 42°. Depth consistency is poor at 36° and improves smoothly to a tight Depth consistency at 42° and then deteriorates smoothly to a poor Depth consistency at 55°. Remarkably, these trends in Depth consistency hold...
true across all quartiles of the data as you can see in the full range of shots shown in Figure 7A. The 2\textsuperscript{nd} and 3\textsuperscript{rd} quartile ranges can be inspected more closely in Figure 7B.

![Boxplot of Depth values](image1)

**Figure 7:** A) A boxplot showing the distribution of Depth values for each Angle value across all measured shots. The box denotes the middle two quartiles of the distribution and the wings represent the 5\textsuperscript{th} and 95\textsuperscript{th} percentiles. B) Same data with expanded Y-axis to allow closer inspection of the Depth median and Depth consistency across Angle values.

Figures 7A and 7B are not typical of normal distributions associated with human performance, so how can these unusual patterns be explained? The laws of physics hold the answer. Projectile physics calculates, for example, that players shooting a 3-point shot with constant release velocity will maximize their shot depth at a 42° Angle. If they shoot the ball higher than 42°, Depth will decrease. If they shoot the ball flatter than 42°, Depth will also decrease. And the further the Angle deviates from 42°, the shorter the Depth. This principle of physics is described further in the Appendix.

Although a 42° Angle value optimizes Depth consistency, it is not the optimal Angle to maximize shooting percentage. This will be made clear in the next section which defines the GMZ.

5. **Defining the Guaranteed Make Zone (GMZ) as a function of shooter factors**

We next explored how shooter factors interact to explain high percentage shooting. We found that the family of shots for each Angle value has a different scoring percentage at each Depth value. For example, a shot with a high Angle (high arc) will be less likely to hit the front rim than a low Angle shot (flat arc), so Angle value influences whether a particular Depth value will score. As an illustrative example, we selected all shots with an Angle value of 45°, the most common in the population, and then produced the distribution of shooting percentages for each Depth value at the 45° Angle (Figure 8A). As a starting point, we only considered straight shots with a Left-Right value of -2” to +2”. Straight shots with an Angle of 45° which cross the hoop plane at a Depth value of 7” to 14” are guaranteed to score either by swishing or by hitting the back of the rim and deflecting down. These depths are shown in the yellow band. The shooting percentages in the yellow zone do not quite reach 100% due to situations such as shots colliding with other shots during practice sessions or sensors which had not been upgraded to the latest version at the time the shot was taken. For our data analysis, we will consider all depths with greater than 90% shooting percentage to be defined as the Guaranteed Make Zone (GMZ). Depths outside the GMZ have a low
or near zero chance of scoring because they will hit the rim and bounce up or sideways. For purposes of this analysis, the percentage of shots in the GMZ to total shots for a player are a close approximation to their shooting percentage. The GMZ for the 45° Angle family in Figure 8A is reflected in Figure 8B as shown by the bracket.

Each Angle family has a slightly different distribution of make percentages for each Depth value. By creating a figure similar to Figure 8A for all Angles from 35 degrees to 55 degrees, we created a GMZ map for straight shots across all angles and depths (Figure 8B). Note that the GMZ is narrower at 35°, gets wider near 45°, and then narrows again at 55°.

Figure 8: A) We show the make percentage at each Depth value for straight shots with an Angle value of 45°. B) We show the Guaranteed Make Zone (GMZ) for straight shots across all Angle values.

But not all shots are straight, and the Left-Right value also impacts the size and shape of the GMZ. To demonstrate this phenomenon, we split the hoop into three sections according to the Left-Right value (Figure 9A). The yellow band represents straight shots within 2” of the centerline of the hoop. The green bands represent shots that are nearly straight, between 2” and 4” from the centerline of the hoop. Finally, the blue bands represent shots that enter the plane of the hoop between 4” and 5” inches from the centerline of the hoop. As the Left-Right value deviates from straight, the GMZ dramatically shrinks (Figure 9B-D). Few shooters consistently shoot only in the yellow zone, making it even more important that shooters consistently shoot the correct Depth value for their shots in order to increase shooting percentage.

Figure 9: A) We show three left-right regions in the hoop – 0 to +/-2, +/-2 to +/-4 and +/-4 to +/-5. B) The GMZ for 0 to +/-2. C) The GMZ for +/-2 to +/-4. D) The GMZ for +/-4 to +/-5.
6. Overlaying the GMZ on factor figures to draw insights on actionable ways to increase 3-point shooting percentage

A key objective of this paper is to improve 3-point shooting percentage by identifying actionable training approaches. Each of the four insights described below can be implemented using instant verbal feedback training techniques, and the results of that training can be verified by measuring improvement of the factors in game situations.

When the GMZ from Figures 9B and 9D overlay the Angle-Depth chart from Figure 7B, it is apparent there are many reasons players miss shots. The actionable factors for improving 3-point percentage are Left-Right consistency, Angle median, Angle consistency and Depth median. Although Depth consistency would appear to be an important factor to increasing shooting percentage, it is not directly actionable because it is influenced by too many other factors including Angle median and Angle consistency. Thus, Depth consistency, like shooting percentage, is considered an outcome and is not included in actionable factors for predicting or improving shooting percentage.

6.1 Left-Right consistency is key to widening the GMZ to increase shooting percentage

Figure 3A shows that of the 1.1 million shots, only 35% fall within the -2” to +2” Left-Right band encompassed in the yellow GMZ of Figure 10A. 27% of the shots fall 5” or more away from the centerline creating a GMZ with few or zero made shot opportunities as shown by the blue GMZ in Figure 10B. The first key to increasing shooting percentage is to shoot straight so a player is operating in the yellow GMZ of Figure 10A rather than the blue GMZ of Figure 10B.

Figure 10: We show the distribution of Depth values per entry Angle A) overlaid with the yellow GMZ of straight shots and B) overlaid with the blue GMZ of +/- 4 to +/- 5 Left-Right shots.
6.2 Angle median is necessary to have the right balance of Depth consistency and size of the GMZ to increase shooting percentage

Of the 162 shooters shown in Figure 5A, 69% had an Angle median between 43° and 47°. Having an Angle median in this range is important because of Angle median’s influence on Depth consistency as described in section 4.2 and the Appendix as well as Angle median’s influence on the size of the GMZ as described in section 5. A 42° Angle median will have the tightest depth control, but a narrower GMZ. A 47° Angle median will have less depth control, but a wider GMZ. Players with Angle median in the range between 43° and 47° are most able to fit their shots into the GMZ and thus have a similar shooting percentage all other factors held equal. A 45° Angle median is the best place to practice, especially for less proficient shooters.

6.3 Angle consistency is necessary to tighten Depth consistency so more shots can be in the GMZ and increase shooting percentage

As shown in Figure 6, Angle consistency is the second factor necessary to tighten Depth consistency. Players who can minimize their angle variance while maintaining constant power will shoot their shots at a more consistent depth.

6.4 Depth median is necessary to center an athlete’s shots in the GMZ to increase shooting percentage

Figures 10A and 10B show that an ideal Depth median is about 11.0” because that will center the range of shot depths over the GMZ to maximize shooting percentage. But Figure 4A shows that the population of 1.1 million shots have a Depth median of 9.3” which is 1.7” short of the ideal.

Players shoot a Depth median below the ideal of 11” partly because laws of physics will cause the human error in Angle to shorten the median Depth (see Appendix). The laws of physics cannot be changed with improved shooting mechanics or improved muscle memory; we observe that shot Depth median is a consistent problem across all skill levels from the NBA to high school. The Depth median problem can be minimized by shooting an Angle median in the mid-40’s (see 6.2) with great Angle consistency (see 6.3) and aiming for 11” instead of simply a swish.

7. The four actionable shooter factors can predict shooting percentage

Next, we sought to determine the capacity of the four actionable shooter factors to predict player shooting percentage. Since we previously showed Depth consistency is not actionable because it is a function of other factors such as Angle consistency and Angle median, we exclude Depth consistency as a factor. Furthermore, we also exclude Left-

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Figure 11: A) We show the correlation between the actual shooting percentage and the predicted shooting percentage for the players in our test set.

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Right median because shooters can generally self-correct for this attribute. Thus, our remaining factors are Angle median, Depth median, Angle consistency and Left-Right consistency. We split the 162 players with more than 200 recorded shots into a training and a testing group. We used cross validation to train and optimize the parameters of a gradient boosting regressor. Our resulting model had 100 estimators and a learning rate of 0.1. Our predictions on our test set received a Spearman correlation of 0.89 between the actual shooting percentage of the players and the predicted shooting percentage; this resulted in an error of 1.5% (Figure 11).

8. Player case studies

8.1 Cluster players on improvement potential

All four of these actionable shooter factors can be improved in order to increase shooting percentage. However, each player has a unique baseline of these factors, so improvement of a specific factor holds different potential for each player. To quantify improvement potential for each factor, we separated players into two categories – ‘Proficient’ and ‘Not Proficient’ – for each of the four actionable factors. We grouped the players according to these factor states (Figure 12). There are 15 groups. The group at the top is where all actionable factors are ‘Not Proficient’, and the group at the bottom is where all actionable factors are ‘Proficient’. Note the median shooting percentage in the cluster with all ‘Not Proficient’ is 39% and the median shooting percentage in the cluster with all ‘Proficient’ is 68%. Next, we take a closer look at two case studies to understand the improvement potential of players with specific factor expressions.

8.2 Player case studies

Player X is a 47% three-point shooter during practice. She is ‘Proficient’ in Angle median and Depth median and ‘Not Proficient’ in Left-Right consistency and Angle consistency. Her shot distribution is centered on her GMZ, yet her GMZ is severely narrowed by her Left-Right inconsistency (Figure 13A). By focusing on her Left-Right consistency, she will expand her GMZ and...
make many more shots regardless of her Angle and Depth inconsistency (Figure 13B). Through improvement in Left-Right consistency alone, she can expect to reach a 3-point shooting percentage of 56% during practice once she reaches a level of ‘proficiency’, an overall improvement of 9%. Other players clustered in the same category as her should also focus on Left-Right consistency to optimize their path to improved shooting percentage.

Player Y is a 64% 3-point shooter during practice. He is proficient in all categories except for median Depth. While one can always expect to increase one’s shooting percentage with improved Angle consistency and Left-Right consistency, Player Y has straightforward improvement potential if he focuses first on improving his Depth median. He is currently shooting with a Depth median of 8.5 inches (Figure 14A). If he increases his Depth median by 2 inches to 10.5 inches, he can expect to become a 70% three-point shooter in practice – an overall improvement of 6%. Due to few examples of players shooting the optimal depth, this percentage increase was determined by sampling Player Y’s shot population to simulate the deeper Depth median. This increase in Depth median will create a much better overlap between his shot distribution and GMZ (Figure 14B).

Figure 13: A) We show the current shot distribution and current GMZ of Player X. B) We show improved, wider GMZ of Player X if Left-Right consistency is improved to ‘proficient’ resulting in more shots in the GMZ and a higher shooting percentage.

Figure 14: A) We show the current shot distribution of Player Y where Depth median is not centered in the GMZ. B) We show the potential shot distribution of Player Y if his median Depth is improved by centering in the GMZ.
8.3 Practice performance translates to game performance for individual players

The Noah shooting system collects shot data (6 ball dynamics factors in addition to player identification, shot location and make-miss) for both practices and games. Currently, the game data is limited, but the initial data shows strong correlation between practice performance and game performance for seven individual shooters with sufficient game data available. A few examples: a strong correlation exists between practice 3-point shooting percentage and game catch and shoot 3-point shooting percentage (Spearman rho=-0.82, p-value = 0.02); a good correlation occurs between practice Left-Right consistency and game catch and shoot 3-point shooting percentage (Spearman rho=-0.78, p-value = 0.03); and a good correlation occurs between practice Depth consistency and game catch and shoot 3-point shooting percentage (Spearman rho=-0.78, p-value = 0.03). As we collect more game 3-point data on the four actionable player factors (Angle median, Angle consistency, Depth median and Left-Right consistency), we expect to see a wide range of additional correlations which will instruct practice methodologies to maximize 3-point shooting performance for individuals with specific factor expressions.

9. Conclusions

Increasing 3-point shooting percentage is fundamental to winning games, but the NBA 3-point shooting percentage has been stuck at 35% over the past 20 years. Over those 20 years, the fundamental 3-point training method has not changed; players take a lot of 3-point shots with good shot mechanics and count the number of makes and misses. In this paper, we explored 3 shot attributes and 6 shooter factors which give much greater understanding into why shots are made or missed. This understanding is supported by shot data from 1.1 million 3-point shots. With the availability of real-time, verbal feedback on Left-Right, Angle and Depth, individual athletes are now able to focus their workouts on the specific aspects that limit their game 3-point shooting percentage.

Using advanced computer vision and machine-learning techniques, there is an exponential increase in our ability to collect and analyze 3-point shot data using the 6 shooter factors. This newly available data will provide opportunity for many high potential analytics research efforts including:

• What training methodologies are most efficient in improving shooting percentage for each cluster of shooter factors? What is the shooting percentage improvement pace for specific clusters using specific training methodologies?

• Most players have different expressions of the 6 shooter factors for a “catch and shoot” compared to a “dribble left and shoot” or a “dribble step back and shoot”. What are the most efficient training methods for players to increase shooting percentage across all shot types?

In the coming years, there is a large opportunity to complete meaningful longitudinal studies since 2500 middle school and high school athletes are already training regularly with instant verbal feedback on Left-Right, Angle and Depth.
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References

Appendix

Physics behind Angle and Depth interaction

As mentioned in Section 4.2, Figures 7A and 7B are not typical of normal distributions associated with human performance. In these figures, laws of physics dominate shooting skill in determining Depth consistency results. Projectile physics calculates, for example, that players shooting a 3-point shot from an 8 foot release height with constant release velocity will maximize their shot depth at a 42° Angle (Figure 13). If they release the ball either steeper or shallower than 42°, Depth will decrease, a counterintuitive outcome for most observers without a physics background.

To demonstrate this concept, we consider three fictional 3-point shooters:

Flat shooter #1 shoots 11 shots with constant release velocity in one degree Angle increments from 33° to 43° with a median Angle of 38°. At 38° the Depth is 8.8”. At 33° the Depth is 0.5”, and at 43° the Depth is 11.0”. The Depth range is 10.5”.

High shooter #2 shoots 11 shots with constant release velocity in one degree Angle increments from 42° to 52° with a median Angle of 47°. At 47° the Depth is 8.2”. At 42° the Depth is 11.1”, and at 52° the Depth is -1.1”. The Depth range is 12.2”.

Medium Angle shooter #3 shoots 11 shots with constant release velocity in one degree Angle increments from 37° to 47° with a median Angle of 42°. At 42° the Depth is 11.1”. At 37° the Depth is 7.6”, and at 47° the Depth is 8.2”. The Depth range is 3.5”.

In this simplified example, the Depth range (associated with Depth consistency) for the 42° medium angle shooter is far better than the Depth range for either the flat shooter or the high shooter.