



# A PageRank Model for Player Performance Assessment in Basketball, Soccer and Hockey

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## Abstract

In the sports of soccer, hockey and basketball the most commonly used statistics for player performance assessment are divided into two categories: offensive statistics and defensive statistics. However, qualitative assessments of playmaking (for example, making “smart” passes) are difficult to quantify. It would be advantageous to have available a single statistic that can emphasize the flow of a game, rewarding those players who initiate and contribute to successful plays more. In this paper we will examine a model based on Google's PageRank. Other papers have explored ranking teams, coaches, and captains but here we construct ratings and rankings for individual members on both teams in a game that emphasize initiating and partaking in successful plays and forcing defensive turnovers.

For a soccer/hockey/basketball game, our model assigns a node for each of the  $n$  players who play in the game and a “goal node”. Arcs between player nodes indicate a pass in the *reverse* order (turnovers are dealt with separately). Every sport-specific situation (fouls, out-of-bounds, play-stoppages, turnovers, missed shots, defensive plays) is addressed, tailored for each sport. As well, some additional arcs are added in to ensure that the associated Markov chain transition matrix is primitive (some power of the matrix has all positive entries) and hence there is a unique PageRank vector, which is used to rate and rank the players of the game.

To illustrate the model, data was taken from nine NBA games played between 2014 and 2016. The application shows that this model does indeed provide the type of comprehensive statistic described in the introductory paragraph. Many of the top-ranked players (in the model) in a given game have some of the most impressive traditional stat-lines. However, from the model there are surprises where some players who have impressive stat-lines have lower ranks, and others, who have less impressive stat-lines have higher ranks.

Overall, the model provides an alternate tool for player assessment in soccer, basketball and hockey. The model's ranking and ratings reflect more the flow of the game compared to traditional sports statistics.



## 1. Background

Google PageRank was created as the backbone to what is now the most influential search engine ever created [7]. Its purpose is to rank the importance of web pages when a user makes a query. The foundations of PageRank lie in Markov chain theory (see, for example [9]): given a finite set of states  $S = \{s_1, \dots, s_n\}$ , let  $t_{i,j}$  be the probability of moving from state  $s_i$  to state  $s_j$  from time  $k$  to  $k + 1$  (note that  $t_{i,j}$  is independent of  $k$ ). Let  $T = [t_{i,j}]$  denote the *transition matrix* of the Markov chain. Provided that the matrix  $T$  is *primitive* (i.e.  $T^m > 0$  for some positive integer  $m$ ), there is a unique *stationary vector*  $\mathbf{v}$ , such that  $\mathbf{v}^t \mathbf{1} = 1$  and  $T^t \mathbf{v} = \mathbf{v}$ , where  $\mathbf{1}$  is the vector in  $\mathbb{R}^n$  consisting of all 1's (that is,  $\mathbf{v}$  is an eigenvector of  $T^t$  with eigenvalue 1 whose nonnegative entries sum to 1). We call such a vector the *PageRank vector* of the Markov chain. The lack of primitivity in Google's Markov model in general requires some alteration to the transition matrix in that case. Each component of the PageRank vector is thought of as the *rank* of that state (and an ordering of the states is derived from these values).

There is a natural way to construct a Markov chain from a (finite) directed graph and its adjacency matrix. The states of the chain are the nodes of the graph. If there are  $n_{i,j}$  ( $= A_{i,j}$ ) arcs from node  $i$  to node  $j$  and node  $i$  has a total of  $n_i$  outgoing arcs, then  $T_{i,j} = \frac{n_{i,j}}{n_i}$ . For a Markov chain derived from a directed graph, primitivity of the Markov chain corresponds to the existence of a positive integer  $k$  such that there is a walk of length  $k$  between any two nodes. In such a case there is a unique stationary vector for the associated Markov chain, which can be calculated from a linear system (see [7] for more details). We remark that it has been observed that the PageRank vector is fairly insensitive to small changes in the network involving only lowly ranked nodes [4, 5].

Previous applications of PageRank to sports metrics usually address ranking either teams [1, 2, 10], coaches [8], or individual players on various teams [10, 11]. A number of models have relied on underlying graph networks of games in their respective sports. In [12] a directed graph representing the passes between the starting players on an individual soccer team was constructed and a PageRank vector was computed to highlight who were the most important players on the team based on ball reception; the network as well was analyzed to determine the strategies and weak points of each team. In [6] every team has a corresponding weighted directed graph representing passes and two additional nodes representing the other team's goal and missed shots. Cricket batsmen and bowlers that face each other on separate teams are compared using a basic model much like the one that compares teams in the "win-loss" PageRank method for ranking teams [11]. A weighted directed graph for players across many basketball teams is created in [13] where arcs exist only between players who played on the court together on the same team at some point and the weight of these arcs corresponds to how effective they were in playing together. Finally, in [3] a PageRank network is created for each individual soccer team, where arcs indicate passes between players.

None of these models allow for the effective comparison of any two players playing in the same game together (or even in different games), possibly of different positions or teams, using a PageRank method. More significantly, they do not emphasize playmaking ability, as opposed to pure offensive statistics, and that is exactly what we plan to do.



## 2. The Model

We create a directed graph (which we abbreviate as a *digraph*) representing the progression of play during a particular game – whether soccer, basketball or hockey. For each of the  $n$  players in the game there is a node, and the digraph contains one additional *goal node* (while our implementation of a goal node is not unique, the use of a “missed shot” node in [6] is redundant when we have both teams that are competing against each other in a game represented on the same digraph). Since we desire a model that values playmaking, we must reverse the direction of most arcs that would result from a straightforward progression of play (such an idea was raised but not deeply pursued in a multi-team setting in [3]). For player  $i$  and player  $j$  (represented by node  $i$  and node  $j$ , respectively) on the same team, whenever player  $i$  passes to player  $j$  we draw an arc from node  $j$  to node  $i$ . However, if players  $i$  and  $j$  are on separate teams and player  $i$  loses the ball/puck to player  $j$  we draw an arc from node  $i$  to node  $j$ . If player  $i$  scores, the sport specific value of the score will be the number of arcs drawn from the goal node to node  $i$  (for example an NBA 3-pointer would result in three arcs). All of our choices for arc direction ensures the flow of rank rewards playmaking. There will be more game specific arcs, to be discussed below.

We initialize the digraph for a given game as follows. We draw arcs in both directions between each player node and the goal node, and in addition, we draw a loop from the goal node to itself. Outside of goal scoring, no other arcs will be added to or from the goal node. If team 1 has  $n_1$  players and team 2 has  $n_2$  players ( $n_1 + n_2 = n$ ), then in the corresponding game adjacency matrix,  $A$ ,  $A \in M_{n+1, n+1}(\mathbb{R})$ , the first  $n_1$  columns (and rows) of  $T$  represent players on team 1, columns (and rows)  $n_2 + 1$  to  $n$  representing players on team 2, and the  $(n + 1)$ th column and row representing the goal node. Thus, before the game starts,  $A_{n+1, j} = A_{i, n+1} = 1$  for  $1 \leq i, j \leq n + 1$  and  $A_{i, j} = 0$  otherwise. The method of construction of the initial digraph ensures that any two nodes are connected by a path of length exactly two, and hence the corresponding Markov chain transition matrix,  $T$ , will have each entry of  $T^2$  nonnegative, making  $T$  primitive, and therefore the Markov chain associated with the digraph has a unique PageRank vector.

We let  $\mathbf{r} = (r_1, \dots, r_n, r_g)$  be the PageRank vector of the game transition matrix. For a number of reasons, to be listed, we shall rescale the values in the PageRank vector (such a process does not change the induced ordering of the players’ ranks). One immediate issue with the model is the fact that the goal node may have different rank in each game depending on the number of players in the game, thus making the comparison of ranks of players in different games dependent on the rank of the goal node. We can scale the computed rank vector  $\mathbf{r}$  by any scalar, as  $\mathbf{r}$  is an eigenvector of  $T^t$ . We standardize  $r$  by defining the relative rank of player  $i$  in the game by

$$rel_i = 50n \cdot \frac{r_i}{\sum_{j=1}^n r_j} = \frac{50n \cdot r_i}{1 - r_g}$$

The choice of scaling is as follows: the denominator removes the effect of the goal node’s rank, and the numerator ensures (a) that the relative rank is insensitive to the number of players in the game and (b) provides values on a reasonable scale, between 0 and  $50n$  (it is not hard to see that the average relative ranks of all players in a game is 50). The relative ranks can thus be used for meaningful comparison of players in different games.

We now return to how the digraph itself is built up in the three sport specific situations. We



illustrate the process with basketball (the rules for soccer and hockey can be found in the Appendix). Each bullet point is a play “event”, with its subsequent descriptor the corresponding arc(s) to add to the digraph.

Basketball rules of implementation:

- Pass from player  $i$  to player  $j$ .
  - an arc from node  $j$  to node  $i$
- Player  $i$  dispossesses player  $j$ . [This could include cases where player  $i$  does not gain possession of the ball after dispossessing player  $j$  – for example a defensive touch leading to the ball being out of play or deflecting a pass still into play but away from its intended target.]
  - an arc from node  $j$  to node  $I$
- Player  $I$  scores  $n$  points where  $1 \leq n \leq 4$ .
  - $n$  arcs from the goal node to node  $I$
- Player  $I$  shoots when being contested and defended by player  $j$  and misses the net. [Same as player  $j$  dispossessing player  $I$ , play resuming with the rebounding/inbounding player. This case includes the situation where player  $j$  blocks player  $i$ .]
  - an arc from node  $i$  to node  $j$
- Player  $i$  shoots and misses the net under no pressure and the ball is rebounded by player  $j$ . [Same as player  $j$  dispossessing player  $i$ .]
  - an arc from node  $i$  to player  $j$
- Player  $i$  fouls player  $j$  and player  $j$  makes at least one free throw.
  - if player  $j$  makes  $n > 0$  free throws then  $n$  arcs are created from the goal node to node  $j$
- Player  $i$  fouls player  $j$  and player  $j$  makes zero free throws. [Same as player  $j$  dispossessing player  $i$  – it was a “smart” foul.]
  - an arc from node  $j$  to node  $i$
- Any stoppage of play that does not have to do with the game (i.e. a technical foul, fan interference, injury, altercation etc.). [Play is dead.]
  - no arc drawn



- Player  $i$  intercepts a pass from player  $j$ . [Same as player  $i$  dispossessing player  $j$ .]
  - an arc from node  $j$  to node  $i$
  
- Player  $i$  touches the ball without having possession (for example the ball hits player  $i$ , or a “pinball” play).
  - no arc drawn
  
- Any unforced turnover by player  $i$ .
  - no arcs drawn

We illustrate the process with a small example. Suppose that two basketball teams, the Reds and the Blues, are playing against each other in a “3-on-3” match (where all baskets are worth one point). We will denote the players on the Reds by A, B and C, and those on the Blues by D, E and F. Before the game begins we have the setup of the digraph shown in Figure 1 (where G stands for the goal node). In this and subsequent diagrams of digraphs, if there exists more than one arc from node  $x$  to node  $y$  we will draw one arc from node  $x$  to node  $y$  but label it with the number of arcs that exist between the two nodes.

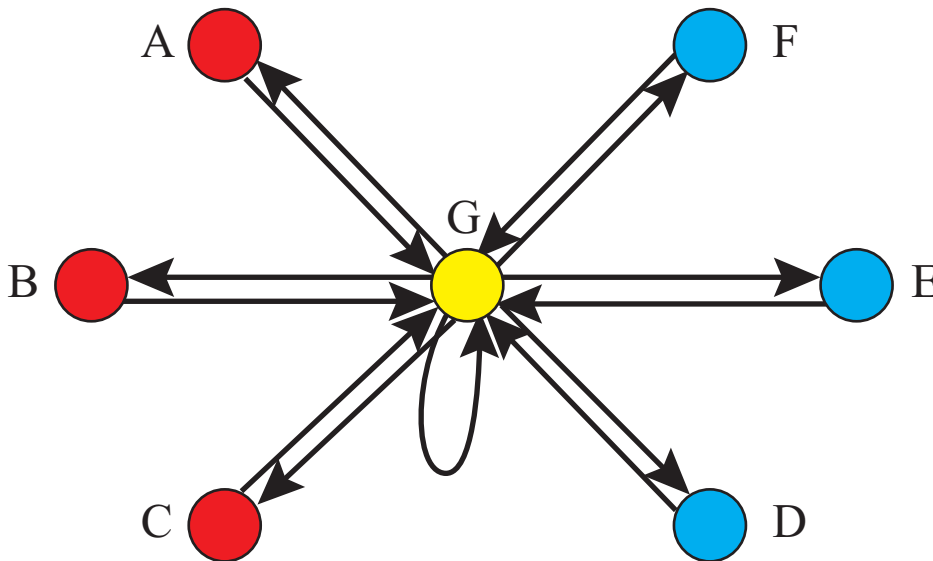


Figure 1: Small example's initial digraph (before play).

Now suppose we have the following sequence of plays in the game: (where “ $\rightarrow$ ” represents the movement of the ball between nodes, and “0” is the end of play sequence symbol):



A→B→A→F→G

D→F→E→F→D→C→B→C→A→C→B→A→G

D→C→A→C→B→A→G

D→F→D→B→C→A→G

D→F→E→F→D→G

A→B→F→G

After these sequences of plays our updated network becomes:

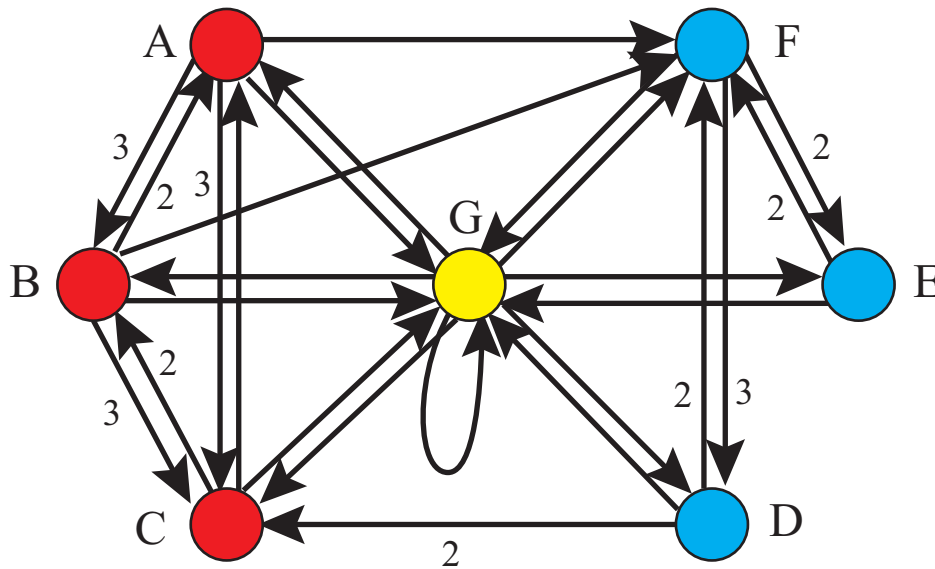


Figure 2: Small example updated

The adjacency matrix of the digraph (whose  $(i, j)$ -th entry is the number of arcs in the digraph from node  $i$  to node  $j$ ) is



$$\begin{bmatrix} 0 & 3 & 3 & 0 & 0 & 1 & 1 \\ 2 & 0 & 3 & 0 & 0 & 1 & 1 \\ 2 & 2 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 3 & 2 & 0 & 1 \\ 4 & 1 & 1 & 2 & 1 & 3 & 1 \end{bmatrix}$$

and so the transition matrix for this game is

$$T = \begin{bmatrix} 0 & 2/7 & 2/5 & 0 & 0 & 0 & 4/13 \\ 3/8 & 0 & 2/5 & 0 & 0 & 0 & 1/13 \\ 3/8 & 3/7 & 0 & 2/5 & 0 & 0 & 1/13 \\ 0 & 0 & 0 & 0 & 0 & 1/2 & 2/13 \\ 0 & 0 & 0 & 0 & 0 & 1/3 & 1/13 \\ 1/8 & 1/7 & 0 & 2/5 & 2/3 & 0 & 3/13 \\ 1/8 & 1/7 & 1/5 & 1/5 & 1/3 & 1/6 & 1/13 \end{bmatrix}$$

The calculated relative ranks of the players in the system are as follows:

Player	Team	Relative Rank
C	Reds	64.66
F	Blues	60.38
A	Reds	58.79
B	Reds	52.39
D	Blues	39.17
E	Blues	24.61

While player A scored the most goals in the game and player F had the same number of total goals and assists, we see that in fact player C, who had only 1 assist and no goals, has the highest rank. However, a cursory examination of the plays clearly shows how integral player C was in the game as a playmaker. The example shows how a player whose contribution to the game might be ignored under the usual stat lines receives their well-deserved acknowledgement under the proposed model.

Before we continue with some experimental results it is natural to ask how this model fits with our intuition of evaluating the performance of athletes. We observe first that the lowest possible



relative rank of any player in a game of  $n$  players is no more than 50, since if the smallest relative rank of any player in the game is more than 50, since if the relative rank of every player were larger than 50, we would contradict the fact that the average relative rank of all players in the game is 50. The following propositions consider the spacing of relative ranks in a game.

*Proposition 2.1.* If there are  $k$  starters (out of  $n$  players) in a given game with a cumulative starter relative rank of  $R$ , then there must exist a bench player with a rank that is at most  $\frac{n(R-50k)}{k(n-k)}$  distance from the rank of some starter.

*Proof.* We first note that the average bench player relative rank must be  $\frac{50n-R}{n-k}$  as there are  $n - k$  bench players and the total sum of all the relative ranks is  $50n$ . Clearly the worst-case scenario occurs if all starters have a rank of  $\frac{R}{k}$  and all bench players have a rank of  $\frac{50n-R}{n-k}$  (as otherwise there would have to be one bench player with relative rank greater than  $\frac{50n-R}{n-k}$  or one starter with rank less than  $\frac{R}{k}$ ). Thus, in this worst-case scenario the distance between any starter and any bench player is  $\frac{R}{k} - \frac{50n-R}{n-k} = \frac{Rn-Rk-50nk+Rk}{k(n-k)} = \frac{n(R-50k)}{k(n-k)}$ . ■

*Proposition 2.2.* If there are  $n$  players in a game then there must exist at least two players who have relative ranks within  $\frac{25n}{n-2}$  of each other.

*Proof.* Clearly 0 is a lower bound on the relative rank of a player in a given game and an upper bound is  $50n$  (the sum of all the relative ranks). Thus, we may partition this range into  $n - 2$  equal length intervals, each of length  $\frac{50n}{n-2}$  using  $n - 1$  of the  $n$  total players. We must have that at least three players must be within (or on the boundary of) one of the  $n - 2$  intervals, meaning that two of their relative ranks must be in the same half interval, making the difference of their relative ranks no more than  $\frac{25n}{n-2}$ . ■

The above two results show that there has to be some bench player of non-negligible importance to the game, and that some players have to have ranks that are “somewhat” close to each other. The first result fits with the widely accepted notion that bench play is a component to the success of a basketball team.

Finally, we may be interested in how the number of goals scored by players on different teams affects their relative ranks. As an illustration, suppose we have only two players,  $P_1$  and  $P_2$ , in the system, each on separate teams, with no interaction between them and  $P_1$  scores  $g_1$  goals and  $P_2$  scores  $g_2$  goals. Then if  $g_1 < g_2$  then the relative rank of  $P_1$  is greater than that of  $P_2$ . This follows as it is clear that the relative rank of  $P_1$  is  $\frac{50nr_g(g_1+1)}{(g_1+g_2+2)(1-r_g)}$  and the relative rank of  $P_2$  is  $\frac{50nr_g(g_2+1)}{(g_1+g_2+2)(1-r_g)}$  where  $r_g$  is the rank of the goal node.





### 3. Sample Data Analysis

We now apply our model to some real-life basketball games. The specific games used were

- Chicago Bulls vs. San Antonio Spurs (November 30th 2015),
- Golden State Warriors vs. Cleveland Cavaliers (January 18th 2016),
- San Antonio Spurs vs. Brooklyn Nets (December 3rd 2014),
- Chicago Bulls vs. Charlotte Hornets (December 3rd 2014),
- Los Angeles Lakers vs. Golden State Warriors (November 1st 2014),
- Toronto Raptors vs. Cleveland Cavaliers (December 9th 2014),
- Golden State Warriors vs. Cleveland Cavaliers (December 25th 2015),
- Chicago Bulls vs. Oklahoma City Thunder (December 25th 2015), and
- Washington Wizards vs. Cleveland Cavaliers (November 26th 2014).

The games are identified by the teams playing, date, score and winning team. In each game, plays were manually analyzed and transcribed, and the resulting transition matrices and PageRank vectors were calculated. The nine tables, in decreasing order, the relative ranks of each of the players (rounded to the nearest hundredth) in all nine NBA basketball games from which data was taken, followed by each players points (P), assists (A), rebounds (R), steals (S), turnovers (T) and field goal percentage (FG%), rounded to the nearest whole number, (in that order) all accessed from nba.com.

Chicago Bulls vs. San Antonio Spurs, November 30th 2015 (Bulls win 92-89)

Player name	Team	Relative Rank	P	A	R	S	T	FG%
Parker	Spurs	103.66	13	9	1	0	0	50
Rose	Bulls	83.68	11	6	4	1	1	29
Duncan	Spurs	74.05	6	3	12	0	2	43
Gasol	Bulls	72.30	18	4	13	1	1	33
Leonard	Spurs	63.62	25	3	8	2	2	77
Aldridge	Spurs	59.99	21	0	12	0	2	55
Noah	Bulls	58.32	8	7	11	0	0	67
Green	Spurs	57.69	9	1	4	2	1	30
Butler	Bulls	49.23	14	3	3	1	5	55
Mirotic	Bulls	46.55	8	2	5	0	2	38
Ginobli	Spurs	44.89	4	2	1	1	1	25
Diaw	Spurs	44.82	5	0	6	0	1	40
Mills	Spurs	40.31	4	1	0	0	0	25
Moore	Bulls	33.42	6	1	2	0	1	50
West	Spurs	31.02	2	1	3	0	0	20
Snell	Bulls	27.48	11	1	6	0	0	80
McDermott	Bulls	24.97	12	0	3	0	0	42
Gibson	Bulls	20.86	4	1	4	1	0	40
Anderson	Spurs	13.13	0	0	0	0	0	0



Golden State Warriors vs. Cleveland Cavaliers, January 18th 2016 (Warriors win 132-98)

Player name	Team	Relative Rank	P	A	R	S	T	FG%
Curry	Warriors	122.91	35	4	5	3	1	67
Green	Warriors	109.34	16	10	7	0	1	50
Dellavedova	Cavaliers	109.26	11	6	1	0	1	50
James	Cavaliers	91.43	16	5	5	1	3	44
Irving	Cavaliers	78.75	8	3	5	0	2	27
Barnes	Warriors	66.94	12	0	2	0	2	50
Livingston	Warriors	62.80	4	0	2	0	2	67
Love	Cavaliers	60.54	3	2	6	0	1	20
Iguodala	Warriors	60.24	20	5	3	0	1	88
Bogut	Warriors	58.61	4	0	6	0	0	67
Varejao	Cavaliers	50.63	5	3	4	1	1	50
Shumpert	Cavaliers	39.84	10	0	2	0	3	67
Barbosa	Warriors	39.52	8	4	1	2	0	50
Thompson	Warriors	39.05	15	2	1	1	1	45
Clark	Warriors	38.24	6	2	2	0	0	29
Ezeli	Warriors	29.79	4	0	2	0	2	67
Mozgov	Cavaliers	28.84	6	3	0	0	1	50
Smith	Cavaliers	26.99	14	1	2	1	1	67
Thompson	Cavaliers	23.73	2	0	2	0	0	0
J Thompson	Warriors	21.89	1	1	3	0	0	1
Cunningham	Cavaliers	20.39	9	1	3	0	0	60
Jefferson	Cavaliers	18.47	6	0	3	0	1	100
Jones	Cavaliers	17.43	8	1	0	0	1	60
Speights	Warriors	17.22	4	0	1	0	0	25
Rush	Warriors	17.17	3	1	3	0	0	33



San Antonio Spurs vs. Brooklyn Nets, December 3rd 2014 (Nets win 95-93)

Player name	Team	Relative Rank	P	A	R	S	T	FG%
Williams	Nets	111.40	17	9	3	0	2	40
Teletovic	Nets	90.99	26	2	15	0	0	69
Duncan	Spurs	81.11	14	1	17	0	2	28
Parker	Spurs	79.92	9	6	1	0	3	50
Ginobli	Spurs	68.23	15	5	6	1	0	46
Green	Spurs	65.77	20	2	10	2	0	50
Lopez	Nets	62.01	16	3	16	0	0	35
Leonard	Spurs	56.59	12	1	13	1	0	25
Joseph	Spurs	55.32	7	3	3	0	0	38
Johnson	Nets	53.48	8	2	5	1	1	25
Jack	Nets	47.11	8	3	1	0	2	40
Diaw	Spurs	45.72	0	3	2	0	2	0
Bonner	Spurs	35.93	7	0	1	0	0	30
Bogdanovic	Nets	32.72	14	0	8	0	2	50
Baynes	Spurs	17.78	4	1	4	1	0	40
Anderson	Nets	14.49	2	1	1	0	2	20
Belinelli	Spurs	11.89	5	1	1	0	1	67
Jordan	Nets	9.83	2	0	2	1	0	100
Plumlee	Nets	9.73	2	0	3	0	1	33

Chicago Bulls vs. Charlotte Hornets, December 3rd 2014 (Bulls win 102-95)

Player name	Team	Relative Rank	P	A	R	S	T	FG%
Walker	Hornets	110.08	23	4	5	1	0	39
Rose	Bulls	85.77	15	5	2	0	2	42
Gasol	Bulls	84.94	19	3	15	0	2	37
Noah	Bulls	77.80	14	7	10	1	2	67
Mirotic	Bulls	73.59	11	1	2	0	1	50
Stephenson	Hornets	64.26	20	4	8	1	4	50
Zeller	Hornets	63.21	12	2	8	0	0	45
Williams	Hornets	59.65	6	0	3	1	0	40
Butler	Bulls	52.88	15	5	2	2	1	45
Brooks	Bulls	51.90	7	3	3	0	2	43
Hinrich	Bulls	48.90	12	2	3	0	1	44
Roberts	Hornets	37.93	3	3	1	0	0	13
Jefferson	Hornets	35.36	13	2	7	0	0	38
Dunleavy	Bulls	32.20	9	0	1	1	0	60
Henderson	Hornets	30.55	10	1	4	0	1	50
Hairston	Hornets	14.67	4	1	2	2	0	14
Snell	Bulls	13.54	0	1	1	0	0	0
Biyombo	Hornets	12.64	4	0	4	0	0	50



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Pargo	Hornets	0.14	0	0	0	0	0	0
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Los Angeles Lakers vs. Golden State Warriors, November 1st 2014 (Warriors win 127-104)

Player name	Team	Relative Rank	P	A	R	S	T	FG%
Curry	Warriors	132.74	31	10	5	3	2	53
Lin	Lakers	94.24	6	6	4	1	5	0
Green	Warriors	89.61	9	1	5	1	1	33
Iguodala	Warriors	84.83	9	6	4	2	4	50
Bogut	Warriors	79.29	6	3	10	1	5	30
Bryant	Lakers	76.87	28	1	6	2	7	43
Hill	Lakers	74.27	23	4	5	0	2	71
Price	Lakers	64.48	1	6	4	2	2	0
Davis	Lakers	59.39	13	2	6	1	1	71
Barnes	Warriors	53.12	15	3	4	1	1	83
Thompson	Warriors	50.27	41	2	5	0	1	78
Livingston	Warriors	42.72	2	1	2	1	1	50
Boozer	Lakers	38.93	9	1	4	0	0	44
Ezeli	Warriors	37.05	3	1	4	0	2	100
Barbosa	Warriors	34.37	9	3	1	1	3	50
Johnson	Lakers	33.40	15	0	4	0	1	67
Ellington	Lakers	20.53	2	1	4	1	1	50
Speights	Warriors	14.58	2	0	3	0	0	50
Sacre	Lakers	7.69	4	0	1	0	2	50
Clarkson	Lakers	5.52	3	0	1	2	1	20
Henry	Lakers	4.95	0	0	0	0	0	0
Holiday	Warriors	1.18	0	0	0	0	0	0



Toronto Raptors vs. Cleveland Cavaliers, December 9th 2014 (Cavaliers win 105-101)

Player name	Team	Relative Rank	P	A	R	S	T	FG%
Lowry	Raptors	123.55	16	14	4	1	0	33
Irving	Cavaliers	118.30	13	10	1	2	2	42
James	Cavaliers	95.39	35	4	2	2	2	57
Love	Cavaliers	70.70	17	4	9	0	3	40
Valanciunas	Raptors	65.34	18	0	15	0	3	86
Dellavedova	Cavaliers	59.32	6	5	3	0	0	50
Patterson	Raptors	47.36	12	1	4	0	1	71
Thompson	Cavaliers	44.27	8	0	8	0	1	60
Williams	Raptors	43.97	6	4	1	0	1	25
A. Johnson	Raptors	41.76	10	2	2	0	2	50
Ross	Raptors	37.68	18	1	3	0	5	62
Vasquez	Raptors	33.11	3	2	0	0	1	33
Fields	Raptors	32.38	4	2	1	2	1	100
Varejao	Cavaliers	32.15	8	1	6	0	1	40
Waiters	Cavaliers	29.92	18	2	1	0	1	70
J. Johnson	Raptors	24.98	12	0	4	1	1	46
Marion	Cavaliers	23.12	0	0	1	0	1	0
Jones	Cavaliers	20.41	0	1	0	0	0	0
Hayes	Raptors	6.30	2	0	1	0	0	100



Golden State Warriors vs. Cleveland Cavaliers, December 25th 2015 (Warriors win 89-83)

Player name	Team	Relative Rank	P	A	R	S	T	FG%
Green	Warriors	140.89	22	7	15	0	4	47
Curry	Warriors	117.29	19	7	7	2	3	40
Love	Cavaliers	106.85	10	4	18	0	1	31
Dellavedova	Cavaliers	98.70	10	1	5	1	1	36
James	Cavaliers	97.45	25	2	9	1	4	38
Iguodala	Warriors	74.47	7	3	2	1	0	17
Irving	Cavaliers	65.18	13	2	3	1	2	27
Thompson	Cavaliers	57.09	8	1	10	1	0	50
Livingston	Warriors	54.35	16	2	3	1	4	89
Thompson	Warriors	47.08	18	1	6	0	1	38
Bogut	Warriors	47.03	4	1	7	0	0	100
Ezeli	Warriors	36.77	3	0	4	0	2	25
Shumpert	Cavaliers	30.38	0	1	4	1	0	0
Smith	Cavaliers	28.77	14	0	1	1	2	44
Rush	Warriors	18.84	0	0	3	1	1	0
Mozgov	Cavaliers	15.40	0	0	3	0	1	0
Clark	Warriors	14.20	0	0	0	1	0	0
Barbosa	Warriors	14.04	0	0	1	0	0	0
McAdoo	Warriors	13.33	0	0	1	0	0	0
Speights	Warriors	10.75	0	0	0	1	1	0
Jones	Cavaliers	5.78	0	0	2	0	0	0
Williams	Cavaliers	5.35	3	1	0	0	0	0



Chicago Bulls vs. Oklahoma City Thunder, December 25th 2015 (Bulls win 105-96) \*last 36.4 seconds of second quarter and first 17 seconds of 3rd quarter were not able to be seen from the source.

Player name	Team	Relative Rank	P	A	R	S	T	FG%
Westbrook	Thunder	125.96	26	8	7	6	6	39
Gasol	Bulls	104.06	21	6	13	0	4	50
Butler	Bulls	94.85	23	4	6	4	3	45
Durant	Thunder	79.36	29	7	9	1	2	52
Rose	Bulls	79.34	19	1	4	0	4	39
Kanter	Thunder	70.19	14	1	13	0	0	50
Gibson	Bulls	62.38	13	2	10	1	1	75
Ibaka	Thunder	58.25	6	0	7	2	2	25
Portis	Bulls	49.98	7	3	5	1	1	38
Hinrich	Bulls	41.15	2	2	0	0	0	50
Adams	Thunder	38.62	3	0	4	0	0	25
Brooks	Bulls	35.67	6	1	4	0	0	50
Mirotic	Bulls	32.04	6	2	7	1	1	20
Augustin	Thunder	28.25	3	1	1	1	2	25
Roberson	Thunder	23.22	2	1	4	0	0	17
McDermott	Bulls	19.31	5	1	3	1	1	29
Morrow	Thunder	18.27	9	0	1	1	0	50
Snell	Bulls	17.27	3	0	1	0	1	25
Walters	Thunder	12.41	2	2	0	1	1	17
Collison	Thunder	9.42	2	0	2	0	0	50



Washington Wizards vs. Cleveland Cavaliers, November 26th 2014 (Cavaliers win 113-87)

Player name	Team	Relative Rank	P	A	R	S	T	FG%
James	Cavaliers	117.45	29	8	10	3	4	50
Irving	Cavaliers	117.00	18	5	1	3	1	47
Wall	Wizards	113.96	6	7	4	0	5	33
Beal	Wizards	62.07	10	2	2	3	1	40
Love	Cavaliers	61.40	21	0	5	0	2	70
Gortat	Wizards	52.63	12	1	2	1	3	50
Thompson	Cavliers	50.89	10	0	1	0	0	100
Waiters	Cavaliers	49.51	15	6	3	2	1	35
Miller	Wizards	48.21	7	6	2	0	0	75
Seraphin	Wizards	47.26	7	3	3	0	2	38
Varejao	Cavaliers	47.03	10	0	7	0	1	100
Humphries	Wizards	45.24	3	1	3	0	1	14
Pierce	Wizards	42.62	15	3	3	0	3	80
Marion	Cavaliers	40.00	6	2	4	2	0	25
Porter Jr.	Wizards	30.72	2	1	2	0	0	25
Cherry	Cavaliers	28.63	2	0	0	2	0	0
Blair	Wizards	23.79	0	0	1	0	1	0
Butler	Wizards	23.11	23	0	1	1	2	60
Amundson	Cavaliers	20.52	0	1	1	0	0	0
Gooden	Wizards	16.66	2	0	3	0	0	50
Harris	Cavaliers	11.30	2	0	2	0	0	50

We observe several general trends from the data that intuitively match what we would expect. For example, out of all the players with relative ranks under 30,  $\frac{48}{55} \approx 87\%$  of them have their sum of points, assists, rebounds and steals no greater than 10, corresponding to a “small” stat line. On the other hand, out of all players who have relative ranks greater than 70,  $\frac{28}{44} \approx 64\%$  have their sum of points, assists, rebounds and steals be at least 25, corresponding to a “large” stat line. In terms of highest ranked positions, seven of the games had a point guard ranked the highest; however, a forward was ranked in the top five ranks in all nine games as well. On average, the starters of both teams owned approximately 7 out of the first 10 highest relative ranks, which fits with our knowledge that at least one bench player must have some significant importance to the game. Out of the nine games sampled, the highest ranked player was on the losing team four times.

Now let us consider some basic trends in certain averages of the relative ranks in the nine games. In the table below all values were rounded to the nearest hundredth, WT stands for winning team, LT stands for losing team and ARR stands for average relative rank.





Game	WT ARR	LT ARR	WT starter ARR	LT starter ARR
1	46.31	53.32	55.85	71.80
2	52.59	47.19	79.37	57.31
3	47.97	51.82	70.11	65.82
4	57.95	42.85	66.72	60.69
5	56.34	43.66	81.00	63.54
6	54.84	45.64	67.93	60.14
7	49.09	51.10	74.23	62.73
8	53.60	46.40	74.53	65.08
9	54.37	46.03	76.58	63.31

Out of the nine games sampled, the winning team had a higher average relative rank than that of the losing team six times. However, the average ranks of starters on the winning team was greater than that for the losing team in eight out of the nine games.

In many cases our intuition of how a strong overall performance by a player should be ranked agreed with the generated relative ranks. For example, consider the cases of LeBron James (Cleveland Cavaliers), Stephen Curry (Golden State Warriors) and Pau Gasol (Chicago Bulls). Each are recognized as being very talented players in the NBA and in each game surveyed in which they played they each had a “strong” stat line, and also a very high relative rank (corresponding to them being excellent playmakers as well). However, there were certainly also some surprises where players with “strong” stat lines did not have large relative ranks. In the Lakers vs. Warriors game, Klay Thompson scored 41 points yet had a very average relative rank of 50.27. In the Wizards vs. Cavaliers game, Butler had 23 points and a rank of about 23.

A general trend that we observe is that out of all the players who had at least 15 points and have a relative rank less than 50, 89% had the sum of their assists, rebounds and steals be under 10, i.e. scoring alone was not generally highly valued by the model. In contrast, out of the 30 players in all games who had at least 5 assists, 28 of them had a relative rank of at least 50 (about 93%), showing a general trend of rewarding passing compared to scoring. There were of course also some overachievers in the sample data; of the top 10 ranks in each game, on average 40% had a sum of their points, assists, rebounds and steals be at most 20, corresponding to what one could call an at most “standard/average” stat line. Specific examples of high ranking players with low stat lines in this model were John Wall (Wizards vs. Cavaliers), Ronnie Price (Lakers vs. Warriors), Jeremy Lin (Lakers vs. Warriors), Tony Parker (Spurs vs. Nets) and Matthew Dellavedova (Cavaliers vs. Warriors and Cavaliers vs. Raptors).

## 4. Conclusion

While the relative ranks of players are comparable between games, in future work we may consider, over a whole season in one sports league, having all players and teams represented by one large digraph with one goal node. This large-scale model could be used as another measure of performance between any athletes in the same league. Note that



adjusting this model for dealing with events such as player trades would not be difficult. In any case, automating the processing of input is a necessary step in scaling the model. Moreover, adjusting the model to encompass more sports that follow similar models to soccer, basketball and hockey, for instance volleyball or water polo, could find useful applications as well.

The PageRank-based model, which we have constructed, appears to be the first of its kind to give a quantifiable measure in which players on different teams and even different games can be ranked and compared, inclusive of their offensive and defensive skills. While the model could serve as a useful mainstream statistic for scouts, coaches, managers and fans, more data analysis is required to see if the model can provide accurate outcome predictions for games (for example, comparing the average relative rank of the starters of each team, prior to that game). The statistics from our proposed model could be used in conjunction with the standard player performance metrics in each sport to help deepen our understanding of who is really influencing the game the most.



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## Appendix

### Soccer rules of implementation:

- Pass from player  $i$  to player  $j$ .
  - an arc from node  $j$  to node  $i$
- Player  $i$  dispossesses player  $j$ . [This could include cases where player  $i$  does not gain possession of the ball after dispossessing player  $j$ ; for example a defensive tackle leading to the ball being out of play or deflecting a pass still into play but away from its intended target.]
  - an arc from node  $j$  to node  $i$
- Player  $i$  scores.
  - an arc from the goal node to node  $i$
- Player  $i$  shoots when being pressured by player  $j$  and misses the net. [Same as player  $j$  dispossessing player  $i$ . This case includes the situation where player  $j$  blocks player  $i$ .]
  - an arc from node  $i$  to node  $j$
- Player  $i$  shoots and misses the net under no pressure. [Play is dead.]
  - no arcs drawn
- Player  $i$  shoots and shot is saved by the goalkeeper, player  $j$ . [Same as player  $j$  dispossessing player  $i$ .]
  - an arc from node  $i$  to node  $j$
- Player  $i$  fouls player  $j$ , not resulting in a goal. [Play is dead.]
  - no arcs drawn
- Player  $i$  fouls player  $j$ , resulting in a penalty or a goal from a free kick. [Smart drawing of a foul by player  $j$ ; scoring from a free kick could include a direct shot, a header or volley from the free kick or a rebound inside the box following the free kick.]
  - an arc from node  $i$  to node  $j$



- Any stoppage of play that does not have to do with the game (i.e. weather, fan interference, injury, altercation etc.). [Play is dead.]
  - no arcs drawn
- Player  $i$  intercepts a pass from player  $j$ . [Same as player  $i$  dispossessing player  $j$ .]
  - an arc from node  $j$  to node  $i$
- Player  $i$  touches the ball without having possession, for example, the ball hits player  $i$ , or a “pinball” play. [Player  $i$  did not have possession.]
  - no arcs drawn
- Any unforced turnover by player  $i$ .
  - no arcs drawn
- Player  $i$  is offside when player  $j$  passes the ball. [Player  $j$  passes the ball to player  $i$  who is in an offside position, so the play ends at player  $i$ .]
  - an arc from node  $i$  to node  $j$

#### Hockey rules of implementation:

- Pass from player  $i$  to player  $j$ .
  - an arc from node  $j$  to node  $i$
- Player  $i$  dispossesses player  $j$ . [This could include cases where player  $i$  does not gain possession of the puck after dispossessing player  $j$ , for example a defensive touch leading to the puck being out of play or deflecting a pass still into play but away from its intended target.]
  - an arc from node  $j$  to node  $i$
- Player  $i$  scores.
  - an arc from the goal node to node  $i$
- Player  $i$  shoots when being defended by player  $j$  and misses the net. [Same as player  $j$  dispossessing player  $i$ ; play resumes with the player that collects the puck after the shot. This case includes the situation where player  $j$  blocks the shot of player  $i$ .]
  - an arc from node  $i$  to node  $j$



- Player  $i$  shoots and the shot is saved by the goalkeeper, player  $j$ . [Same as player  $j$  dispossessing player  $i$ .]
  - an arc from node  $i$  to node  $j$
- Player  $i$  shoots and misses the net under no pressure. [Play is dead.]
  - no arcs drawn
- Player  $i$  draws a penalty from player  $j$  during which no power-play goal is scored. [A “smart” penalty.]
  - an arc from node  $i$  to node  $j$
- Player  $i$  draws a penalty from player  $j$  during which a power-play goal is scored. [A “smart” drawing of a penalty.]
  - an arc from node  $j$  to node  $i$
- Any stoppage of play that does not have to do with the game (i.e. a penalty, fan interference, injury, altercation etc.). [Play is dead.]
  - no arcs drawn
- Player  $i$  intercepts a pass from player  $j$ . [Same as player  $i$  dispossessing player  $j$ .]
  - an arc from node  $j$  to node  $i$
- Player  $i$  touches the puck without having possession (for example the puck hits player  $i$ , or a “pinball” play). [Player  $i$  did not have possession.]
  - no arcs drawn
- Any unforced turnover by player  $i$ .
  - no arcs drawn
- Player  $i$  is offside when player  $j$  passes the puck. [Player  $j$  still passed player  $i$  the puck, the play ended by player  $i$  being in an offside position.]
  - an arc from node  $i$  to node  $j$
- Player  $i$  ices the puck which is touched by player  $j$ . [Same as player  $i$  turning the puck over to player  $j$ .]
  - an arc from node  $i$  to node  $j$