



Optimizing an NBA Team's Approach to Free Agency using a Multiple Choice Knapsack Model

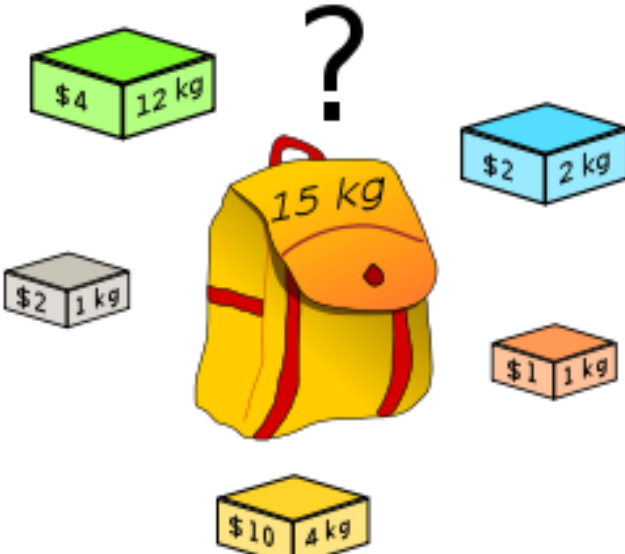
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Knapsack Problem

- Combinational optimization problem
- Items with various weights and benefits are selected to fill a weight constrained knapsack
- The objective is to maximize the utility of the knapsack
- However, the sum of the selected items weight has to be less than the knapsack's fixed capacity

Problem Formulation

$$\max \sum_{i=1}^n \theta_j u_j$$

$$\text{s.t.} \sum_{i=1}^n w_j u_j \leq W$$


Analogy to Free Agency

- The items (u_j) represent the free agents
- The utility (θ_j) represents the benefit of signing the free agent
- The weight constraints (W) represents the team's salary cap space
- The weight of the item (w_j) is the player's annual salary

Complicating Factors

- Player's salaries are dynamic since they are a function of remaining cap space and demand
- Teams are competing to sign free agents from a common pool of players

The Value of Free Agents

- Free agents are characterized by K attributes,
 - $x_{j,k}$ describes player j 's ability of attribute k
- Teams weight the K attributes based on needs,
 - $\lambda_{i,k,t}$ is the weight of k for team i at time t
- The value of player j to team i at time t is given by:

$$\theta_{i,j,t} = \sum_k \lambda_{i,k,t} x_{j,k}$$

GM's Dilemma

- When should GM's offer contracts to coveted players?
- GMs can contract players at the start of free agency at a premium; or
- GMs can wait to acquire the player at a cost which preserves cap space at the risk of losing the player to another team



Free Agency Knapsack Problem

$$\max \sum_{t=\tau}^{T-1} \sum_{j \in J(t)} \theta_{i,j,t} u_{i,j,t}$$

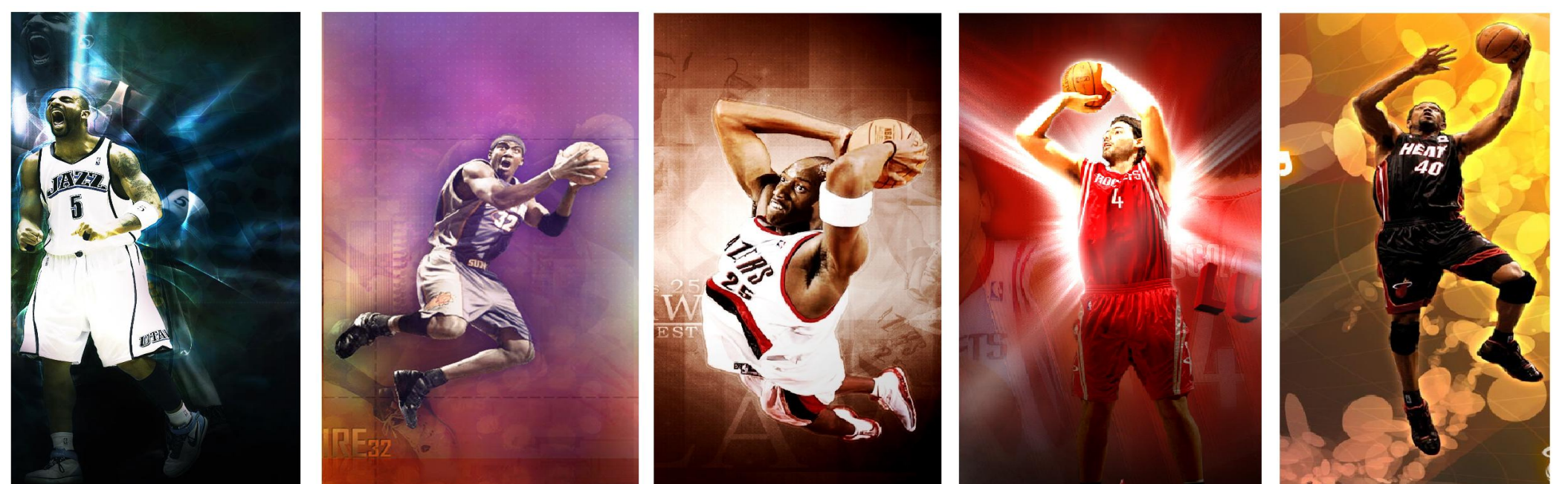
$$\text{s.t.} \sum_{t=\tau}^{T-1} \sum_{j \in J(t)} w_{i,j,t} u_{i,j,t} \leq C_i(\tau), u_{i,j,t} \in \{0,1\}$$

Dynamic Programming Solution

$$f_t(J_t, C_t) = \max_{u_{i,j,t}} \left\{ u_{i,j,t} \left(\theta_{i,j,t} + f_t(J_t - A_j, C_t - w_{i,j,t} B_i) \right) + (1 - u_{i,j,t}) E[f_{t+1}(\bar{J}_t, \bar{C}_t)] \right\}$$

Boundary Conditions:

$$f_t(0, C_t) = f_t(J_t, 0) = 0, \forall t, J_t, C_t, f_T(J_t, C_t) = 0, \forall t, J_t, C_t$$



Control Policy

- GM should offer a contract if the value of the player added to the team is greater than the opportunity cost of the cap space being utilized by the player