



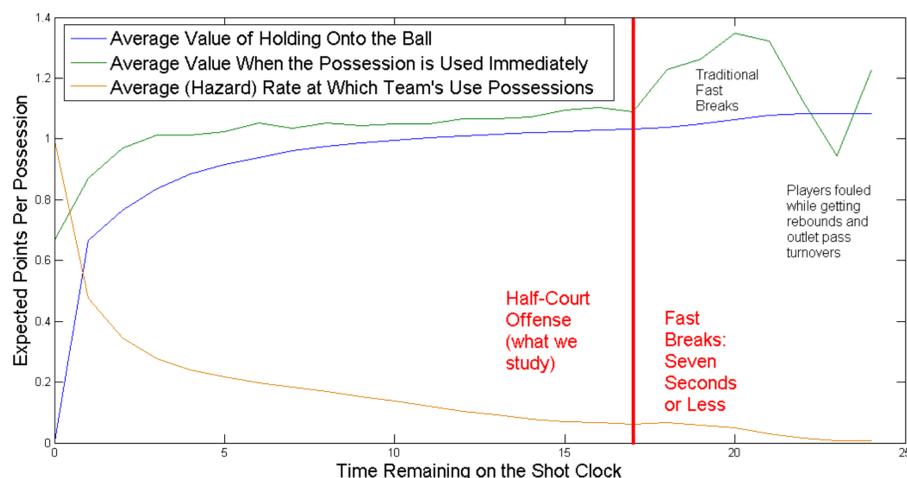
# Dynamic and Allocative Efficiency in NBA Decision Making

Justin Rao & Matthew Goldman/ Yahoo! Labs & UCSD Economics/ jmrao@yahoo-inc.com & mrgoldman@ucsd.edu

## Winning Basketball Games: Modeling the Efficient Use of Possessions in Half-Court

A basketball game consists of a roughly equal number of possessions for each team, where the team who uses their possessions to produce points more efficiently will be the winner. We study the way teams allocate possessions across their players and across the shot clock in an effort to maximize their offensive efficiency within the setting of half-court offense. We link optimality of allocation to a variety of lineup and player characteristics. Additionally, our research gives new insight into the tradeoff players face between usage and efficiency.

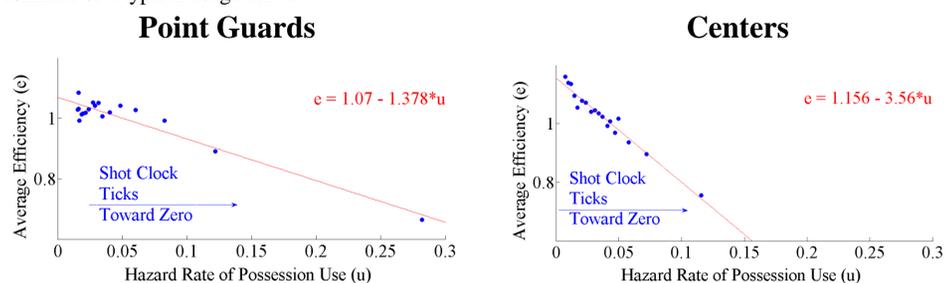
### The Anatomy of a Possession



At every interval of the shot clock, an offensive team must choose between **using the possession immediately** or **holding onto the ball and waiting for another, hopefully better, opportunity**. Opportunities are fundamentally scarce and we see that as the shot clock ticks toward zero, teams must be willing to **use the possession more often** by **accepting lower value scoring opportunities**.

### Usage Curves

Understanding the relationship between how frequently a player shoots and how efficient he is able to be has long been an important issue in the basketball literature. We take the approach of using the shot clock as an *instrument* to identify this relationship. Intuitively, we observe players shooting at a very low hazard rate at the beginning of the shot clock and shooting at a much higher rate at the end of the shot clock - but with a generally lower efficiency. Below we trace out these points for players of different positions as a rough estimate of a typical usage curve.



**Centers face a much steeper tradeoff between their efficiency and usage.** This should be intuitive. Centers rely on their size to obtain highly efficient opportunities around the basket, but may have difficulty creating extra shots. Point Guards are generally more skilled ball handlers and shooters and will have an easier time increasing their usage under the pressure of the shot clock. The slopes of these graphs correspond closely to the usage curves estimated in our paper for an average center and point guard.

### Our Modeling Approach: Shot Opportunity Distributions

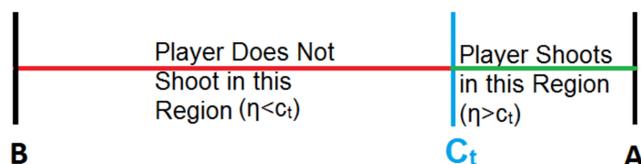
In each one-second period of the shot clock, each player realizes an opportunity to use the possession of value  $\eta$  drawn randomly and uniformly from the interval  $[B, A]$ . With  $t$  seconds left on the shot clock, the player chooses to use the possession if  $\eta > c_t$ , otherwise he waits to the next period of the shot clock.

From such a player, we observe:

$$e_t = (A + c_t) / 2$$

$$u_t = (A - c_t) / (A - B)$$

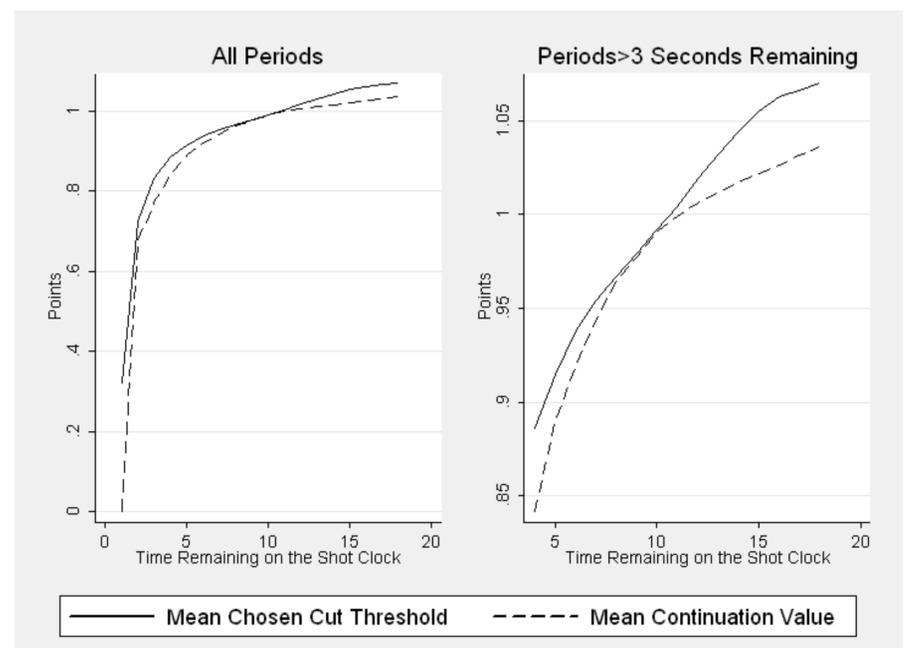
$$de_t/du_t = -(A - B) / 2$$



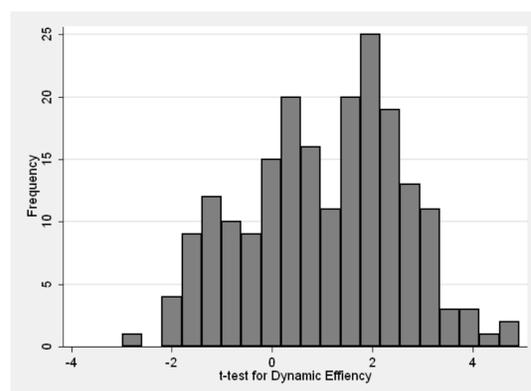
Note that our model implies a linear relationship between efficiency and usage. This is similar to what was observed in the plots above. We fit all the parameters in this model,  $\{A, B, \{c_t\}_{t \in \{0, \dots, 17\}}\}$ , to observed data via the method of Maximum Likelihood Estimation (MLE).

## Results: Do Players Select Shots Optimally?

**Dynamic Efficiency:** The worst shot a player shoots should be of equal value to the continuation value of the possession. In other words, a player should shoot if and only if the value of his shot is higher than the value of holding onto the ball.



The figures above demonstrate a very impressive understanding of Dynamic Efficiency underlying shot selection in the NBA. The average cut threshold ( $c_t$ ), is shown to vary almost perfectly with the average continuation value. However, cut thresholds are slightly too high, indicating that under-shooting is more common than overshooting. In our paper we formally test every player's Dynamic Efficiency. A histogram of the resulting t-statistics is below on the left. A list of the seven most dramatic under and over-shooters is found on the right.



\*t-statistics based on all regular season play from the 2006-07 season through the 2009-10 season. Negative t-statistics correspond to overshooting, positive t-statistics correspond to undershooting.

### Over-shooters

Player	t
Russell Westbrook	-2.98
Tyrus Thomas	-1.98
Lamar Odom	-1.94
Monta Ellis	-1.91
Larry Hughes	-1.8
Drew Gooden	-1.79
Tracy McGrady	-1.77

### Under-shooters

Player	t
Chris Paul	5.34
Brandon Roy	5.13
LeBron James	4.93
Al Jefferson	4.62
Joe Johnson	4.44
Amare Stoudemire	4.06
Vince Carter	4.00

**Statistically significant over-shooting is very rare. Under-shooters are primarily elite players who may, justifiably, be conserving their energy for the long haul of the season.** There is no similar justification for overshooting, so we are not surprised that that it shows up much less commonly and amongst generally lower caliber players.

**Allocative Efficiency:** In each period of the shot clock, all teammates should use the same cutoff for selecting a shot. In other words, Player 1 should never shoot worse shots than Player 2 is passing up.

On the right, is a histogram of the magnitude of deviation from Allocative Efficiency found in all "Three-Man Cores" sharing the court for at least 1,000 possessions.

As you can see, most cores have very little "Spread" and are close to optimal.

In the paper, we discuss the construction of this measure and present **statistically significant evidence that low-experience lineups and high-salary lineups are more likely to allocate shots poorly.**

